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An improved ratio estimator of population mean in two phase sampling scheme

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Abstract— In an attempt to address the problem of efficiency in double sampling strategy, a class of alternative ratio estimator and its generalization is suggested using information on a single auxiliary variable. Members of the suggested class of estimator were obtained by varying some scalars and the bias and mean error of the suggested class of estimators were derived. Analytical and numerical comparison with the usual ratio estimator and some other existing ratio estimators of population mean in double sampling shows that the proposed estimators, though biased is more efficient than the competitor estimators and hence provide a better alternative whenever efficiency is considered.

Index Terms— Bias, Double Sampling, Efficiency, Ratio Estimator, Population Mean.

1. Introduction

The use of information from auxiliary variable has been found to improve the efficiency of estimators. However, where this information is not feasible, double sampling also called two-phase sampling becomes imperative. Double sampling survey is useful for obtaining auxiliary variables for ratio and regression estimation and also for finding information for stratified sampling.

In this sampling scheme, a large sample is selected at the first stage of sampling from which the missing auxiliary information only is obtained after which a second sample is selected in which the variable of interest is measured in addition to the auxiliary information. Double sampling was first advocated by Neyman [1938]. He discovered the importance of two phase sampling techniques while examining the problem of stratification; while the estimation of population mean in two phase sampling for the classical ratio estimator of Cochran [1940] was first advocated by Sukhatme [1962]. Other authors who proposed ratio estimator in double sampling scheme include Hydiroglou and Sarndal [1998], Singh and Vishwakarma [2007], Singh and Espejo [2007], Singh and Choudhury 2012, etc.

The continuous search for better estimators of population mean in double sampling made several author to propose various estimators which were found to be more efficient under some conditions. Such authors include; Solanki and Singh [2013] Singh and Choudhury [2012], Chamu and Singh [2014], Kumar and Vishwakarma [2014], Handique [2012], Kalita and Singi [2013], Sahoo and Singh [2014], Yadov and Kadilar [2013] etc. This paper is a further attempt to present a better method of estimating the population mean in double sampling scheme with desirable properties than some existing ones under certain conditions.

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2. Some Existing Ratio Estimators

Advocated estimator as well as its optimality condition are obtained. This condition is then used to obtain an expression for the Asymptotic Optimal Estimator (AOE), its bias and MSE.

Let $\pi = \{\pi_1, \pi_2, \pi_3, \cdots \pi_N\}$ be a population containing the study and auxiliary variate taking values on the π. Two approaches or cases of estimating the population mean are presented below:

Case I: "A large preliminary sample of size n_1 is selected by simple random sampling without replacement (SRSWOR) from the population of N units and information is obtained on the auxiliary variable alone. A second sub-sample of size n_2, ($n_2 < n_1$) is selected by simple random sampling without replacements (SRSWOR). Information on Y is obtained from the second phase sub-sample".

Case II: A second sample of size n_2 is obtained from the population independent of the first phase sample and information on both the auxiliary and study character are obtained from this sample.

TABLE 1
Some existing ratio estimators in double sampling with MSE

S/N	Estimators	MSE
1.	$\bar{y}$, Sample Mean	$\lambda \bar{Y}^2 C_y^2$
2.	$\bar{y}(\frac{\bar{x}_1}{\bar{x}_2})$, Sukhatme [1962]	$\bar{Y}^2[\lambda C_y^2 + (\lambda - \lambda^1) C_x^2(1-2k)]$
3.	$\bar{y}(\frac{\bar{x}_1}{\bar{x}_2})^{\alpha}$, Srivastava [1970]	$\bar{Y}^2\{\lambda C_y^2 + (\lambda - \lambda^1)\alpha C_x^2(\alpha - 2k)\}$
4.	$\bar{y}\exp[\frac{\bar{x}_1 - \bar{x}_2}{\bar{x}_1 + \bar{x}_2}]$, Singh & Vishwakarma [2007]	$\bar{Y}^2\{\lambda C_y^2 + \frac{(\lambda - \lambda^1)}{4} C_x^2(1-4k)\}$
5	$\bar{y}\exp\ \delta[\frac{\bar{x}_1 - \bar{x}_2}{\bar{x}_1 + \bar{x}_2}]$, Singh et al [2014]	$\bar{Y}^2\{\lambda C_y^2 + \delta\frac{(\lambda - \lambda^1)}{4} C_x^2(\delta - 4k)\}$
6	$\bar{y}[\alpha(\frac{\bar{x}_2}{\bar{X}}) + (1-\alpha)(\frac{\bar{X}}{\bar{x}_1})]$, Singh & Choudhury [2012]	$\bar{Y}^2 \lambda C_y^2 (1-\rho^2)$

3. THE SUGGESTED CLASS OF RATIO ESTIMATORS IN DOUBLE SAMPLING

The suggested class of ratio estimators of the population mean is given as:

$$t_{dr} = \bar{y}\left(\frac{\bar{x}_1 + a\bar{x}_2}{a\bar{x}_1 + \bar{x}_2}\right) \quad \dots (1)$$

where

$\bar{y} = \sum_{i=1}^{n} \frac{y_i}{n}$, the sample mean of the variable of interest obtained from the second phase sample

$\bar{x}_1 = \sum_{i=1}^{n_1} \frac{x_{1i}}{n_1}$, the first phase sample mean of the auxiliary variable

$\bar{x}_2 = \sum_{i=1}^{n_2} \frac{x_{2i}}{n_2}$, the second phase sample mean of the auxiliary variable

$\bar{Y} = \sum_{i=1}^{N} \frac{Y_i}{N}$, the unknown population mean of the study variable Y

$\bar{X} = \sum_{i=1}^{N} \frac{X_i}{N}$, the unknown population mean of the auxiliary variable X

$$\left.\begin{array}{l} e_y = \frac{\bar{y}-\bar{Y}}{\bar{Y}}, \quad e_{x_1} = \frac{\bar{x}_1-\bar{X}}{\bar{X}}, \quad e_{x_2} = \frac{\bar{x}_2-\bar{X}}{\bar{X}} \\ \bar{x}_1 = \bar{X}((1+ e_{x_1}) \\ \bar{x}_2 = \bar{X}(1+ e_{x_2}) \\ \bar{y} = \bar{Y}(1+ e_y) \end{array}\right\} \quad \dots (2)$$

a is a suitably chosen scalar"

Expressing (1) in terms of (2) we have:

$$t_{dr} = \bar{Y}\left(1+e_y\right)(1+u)(1+v)^{-1} \quad . (3)$$

where $u = g_1 e_{x_1} + g_2 e_{x_2}$, $v = g_2 e_{x_1} + g_1 e_{x_2}$, $g_1 = \frac{1}{1+a}$, $g_2 = \frac{a}{1+a}$

Assume that $|v| < 1$, (3) can be expanded as:

$$t_{dr} = \bar{Y}\left(1+e_y\right)(1+u)(1 - \mathrm{v} + \mathrm{v}^2 + \cdots) \quad \dots (4)$$

to the second order of approximation of (4) we obtain

$$t_{dr} = \bar{Y}\left(1 - \mathrm{v} + \mathrm{v}^2 + \mathrm{u} - \mathrm{uv} + e_y - e_y \mathrm{v} + e_y u\right)$$

$$t_{dr} - \bar{Y} = \bar{Y}\{e_y - (g_1 - g_2)e_{x_2} + (g_1 - g_2)e_{x_1} + (g_1^2 - g_1 g_2){e_{x_2}}^2 + (g_2^2 - g_1 g_2){e_{x_1}}^2 - (g_1^2 + g_2^2 - 2g_1 g_2)e_{x_1}e_{x_2} + (g_1 - g_2)e_y e_{x_1} + (g_2 - g_1)e_y e_{x_2} \quad \dots (5)$$

From (5), we obtained the bias for **Case I** as:

$$B(t_{dr}) = E(t_{dr} - \bar{Y}) = \bar{Y}\{(g_1^2 - g_1 g_2)\lambda C_x^2 + (g_2^2 - g_1 g_2)\lambda' C_x^2 - (g_1^2 + g_2^2 - 2g_1 g_2)\lambda' C_x^2 + (g_1 - g_2)\lambda'\rho C_y C_x + (g_2 - g_1)\lambda\rho C_y C_x\} \quad \dots (6)$$

Where $\lambda = \frac{1}{n_2} - \frac{1}{N}$, $\lambda' = \frac{1}{n_1} - \frac{1}{N}$

The MSE is obtained as:

$$\mathrm{MSE}(t_{dr}) = E(t_{dr} - \bar{Y})^2 = \bar{Y}^2 E\{e_y^2 + 2(g_2 - g_1)e_y e_{x_2} + 2(g_1 - g_2)e_y e_{x_1} + (g_2 - g_1)^2 {e_{x_2}}^2 + 2(g_2 - g_1)(g_1 - g_2)e_{x_1}e_{x_2} + (g_1 - g_2)^2 {e_{x_1}}^2\} \quad \dots (7)$$

$$= \bar{Y}^2\left\{\lambda C_y^2 + 2\left(\frac{a-1}{a+1}\right)(\lambda - \lambda')\rho C_y C_x + \left(\frac{a-1}{a+1}\right)^2 (\lambda - \lambda')C_x^2\right\} \quad \dots (8)$$

To obtain the optimum MSE for the suggested estimators, differentiate (8) partially and equate the resulting expression to zero, we get the optimum value of a (i. e the value which minimize the MSE(t_{dr})) as:

$$\hat{a} = \frac{C_x^2 - \rho C_y C_x}{\rho C_y C_x + C_x^2}$$

$$\hat{a} = \frac{1-K}{K+1} \quad , \quad K = \frac{\rho C_y}{C_x} \quad \dots (9)$$

Therefore putting (9) into (8) gives the optimal MSE of the suggested estimator t_{dr} as:

$$MSE_{opt}(t_{dr}) = \bar{Y}^2 C_y^2\left\{\lambda' + \left(\frac{1}{n_2} - \frac{1}{n_1}\right)(1 - \rho^2)\right\} \quad \dots (10)$$

Some members of the suggested estimator with their MSE are shown in Table 2

Remark 1: The optimum MSE of the suggested estimator given in (10) equals the variance of the classical regression estimator in double sampling.

Remark 2: From Table 2, it is shown that the simple random sample per unit mean and the conventional ratio estimator in double sampling are particular cases of the suggested estimator. Also by choosing a suitable value of "a", different ratio estimators in double sampling with their MSE could be obtained as members of the suggested estimator.

TABLE 2
Some members of the suggested estimator with mean square error

S/N	Estimator	a	MSE
1.	$\bar{y}$, sample mean	1	$\lambda\bar{Y}^2 C_y^2$
2.	$\bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)$, Sukhatme (1962)	0	$\bar{Y}^2\{\lambda C_y^2 - 2\rho C_y C_x(\lambda - \lambda') + (\lambda - \lambda')C_x^2\}$
3.	$\bar{y}\left(\frac{\bar{x}_1 + \frac{1}{2}\bar{x}_2}{\frac{1}{2}\bar{x}_1 + \bar{x}_2}\right)$	$\frac{1}{2}$	$\bar{Y}^2\left\{\lambda C_y^2 - \frac{2}{3}(\lambda - \lambda')\rho C_y C_x + \frac{1}{9}(\lambda - \lambda')C_x^2\right\}$
4	$\bar{y}\left(\frac{\bar{x}_1 + (\frac{1-k}{1+k})\bar{x}_2}{(\frac{1-k}{1+k})\bar{x}_1 + \bar{x}_2}\right)$	$\frac{1-k}{1+k}$	$\bar{Y}^2 C_y^2\{\lambda - (\lambda - \lambda')\rho^2\}$

Case II

If the second sample of size n_2 was drawn independently of the preliminary one, then the advocated estimator would still be the same, but the bias and mean square error in this case would be different from the one obtained in the first case. The bias and MSE in this case are derived by setting

$$\mathrm{E}(e_{x_1}e_{x_2}) = E\left(e_y e_{x_2}\right) = 0 \quad \dots (11)$$

in (5) and (7) respectively. Thus from (5) we have

$$\Rightarrow B_2 \quad (t_{dr}) = \bar{Y}\{(g_2^2 - g_1 g_2)\lambda C_x^2 + (g_2^2 - g_1 g_2)\lambda' C_x^2 + (g_1 - g_2)\lambda\rho C_y C_x\} \quad \dots (12)$$

Also, from (7), the mean square error is given as:

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$\mathrm{MSE}_2(t_{dr}) = \bar{Y}^2\mathrm{E}\{e_y{}^2 + 2(g_1 - g_2)e_y e_{x_1} + 2(g_2 - g_1)^2 e_{x_2}{}^2 + (g_1 - g_2)^2 e_{x_1}{}^2\}$

And by applying condition (11) we obtain

$$\mathrm{MSE}_2(t_{dr}) = \bar{Y}^2\{\lambda C_y^2 + 2\left(\frac{a-1}{a+1}\right)\lambda\rho C_y C_x + (\frac{a-1}{a+1})^2(\lambda + \lambda')C_x^2\} \quad \ldots(13)$$

To obtain the optimum MSE, (13) is differentiated with respect to a and resulting expression equate to zero, we get the optimum value of a (i. e the value which minimize the $\mathrm{MSE}_2(t_{dr})$) as:

$$\hat{a} = \frac{\lambda(1-k) + \lambda'}{\lambda(k+1)+\lambda'} \quad \ldots (14)$$

Substituting (14) into (13) gives the optimum MSE as:

$$MSE_{2_{opt}}(t_{dr}) = \bar{Y}^2 C_y^2\{\lambda - \left(\frac{\lambda^2\rho^2}{\lambda+\lambda'}\right)\} \quad \ldots(15)$$

$$= \bar{Y}^2 C_y^2\{\lambda - w\rho^2\}, \quad w = \frac{\lambda^2}{\lambda+\lambda'}$$

Similar members of this class of estimators as expressed in Table 2 can also be obtained in this case, excepts that the optimum MSE's differs sparingly as shown in (13) and (15).

3.1 Efficiency of Comparison

3.1.1: Comparison with Sukhatme [1962] estimator.

Let $\mathrm{MSE}(\bar{y}_{dr})$ denote the mean square error of Sukhatme (1962) estimator, then the suggested estimator at optimum condition is said to be uniformly better than the Sukhatme (1962) estimator, if

Case I

$$\mathrm{MSE}(t_{dr}) < \mathrm{MSE}(\bar{y}_{dr}) \quad \ldots(16)$$

$$\bar{Y}^2 C_y^2\{\lambda - (\lambda - \lambda')\rho^2\} < \bar{Y}^2\left[\lambda C_y^2 + (\lambda - \lambda')(C_x^2 - 2\rho C_y C_x)\right]$$

$$(C_y p - C_x)^2 > 0 \quad \ldots (17)$$

When (16) holds, then the advocated estimator would be more efficient than the Sukhatme (1962) estimator.

Case II

The suggested estimator is more efficient if

$$MSE_2(t_{dr}) < \mathrm{MSE}(\bar{y}_{dr}) \quad \ldots (18)$$

$$\Rightarrow \bar{Y}^2 C_y^2\{\lambda w\rho^2\} < \bar{Y}^2\left[\lambda C_y^2 + (\lambda - \lambda')(C_x^2 - 2\rho C_y C_x)\right]$$

$$\Rightarrow C_y^2 w\rho^2 > \left[(\lambda' - \lambda)(C_x^2 - 2\rho C_y C_x)\right] \quad \ldots(19)$$

When (18) holds, then the advocated estimator would be more efficient than the sukhatme (1962) estimator.

3.1.2 Comparison with Singh and Vishwakarma (2007) estimator

The suggested estimator at optimum condition would be uniformly better than the Singh and Vishwakarma (2007) estimator if

$$\mathrm{MSE}(t_{dr}) < \mathrm{MSE}(\bar{y}_{dr}) \quad \ldots (20)$$

Case I

$$\mathrm{MSE}(t_{dr}) < \mathrm{MSE}(\bar{y}_{dr})$$

$$\bar{Y}^2 C_y^2\{\lambda - (\lambda - \lambda')\rho^2\} < \bar{Y}^2\left[\lambda C_y^2 + \frac{(\lambda - \lambda')}{4}(C_x^2 - 4\rho C_y C_x)\right]$$

$$\Rightarrow (2C_y\,\rho - C_x)^2 > 0 \quad \ldots (21)$$

(25) is always true, it implies that the advocated estimator is always better than the Singh and Vishwakarma (2007) estimator.

Case II

$$MSE_2(t_{dr}) < \mathrm{MSE}(\bar{y}_{dr})$$

$$\bar{Y}^2 C_y^2\{\lambda - w\rho^2\} < \bar{Y}^2\left[\lambda C_y^2 + \frac{1}{4}(\lambda C_x^2 - 4\rho C_y C_x) + \lambda' C_x^2\right]$$

$$\Rightarrow 4wC_y^2\rho^2 > 4\rho C_y C_x - (\lambda + \lambda')C_x^2 \quad \ldots (22)$$

When (22) holds, then the advocated estimator is uniformly better than the Singh and Vishwakarma [2007] in case II.

3.1.3 Comparison with Singh and Choudhury [2012] estimator

Case I

The MSE of both the suggested estimator and Singh and Choudhury [2012] estimators are the same.

Case II

The proposed estimator would be better than Singh and Choudhury [2012] estimator if

$$MSE_2(t_{dr}) < MSE_2(\bar{y}_{SC})$$

$$\Rightarrow \bar{Y}^2 C_y^2\{\lambda - w\rho^2\} < \lambda\bar{Y}^2 C_y^2(1-\rho^2) \quad \ldots (23)$$

3.2 A Generalized Family Of Ratio Estimator Under Two-Phase Sampling.

Motivated by the first proposition, a general family of the ratio estimator is proposed as:

$$t_{drg} = \bar{y}\left(\frac{\bar{x}_1 + a^*\bar{x}_2}{a^*\bar{x}_1 + \bar{x}_2}\right)^\gamma \quad \ldots (24)$$

where γ and a are suitably chosen scalars. Expressing (24) in terms of (2), we have

$$= \bar{Y}(1+e_y)(1+u)^\gamma(1+v)^{-\gamma} \quad \ldots (25)$$

where $u = g_1 e_{x_1} + g_2 e_{x_2}$, $v = g_2 e_{x_1} + g_1 e_{x_2}$, $g_1 = \frac{1}{1+a^*}$, , $g_2 = \frac{a^*}{1+a^*}$

Assuming $|v| < 1$, $(1+v)^{-\gamma}$ can be expanded using Taylor's approximation as

$$t_{drg} = \bar{Y}(1+e_y)(1+u)^\gamma\left(1 - \gamma v + \gamma\left(\frac{\gamma+1}{2}\right)v^2 + \cdots\right)$$

Approximating to second order, we get

$$t_{drg} = \bar{Y}\left\{1 + \gamma(u - v) + \left(\frac{\gamma^2+\gamma}{2}\right)v^2 + \left(\frac{\gamma^2-\gamma}{2}\right)u^2 - \gamma^2 uv + e_y - \gamma e_y(v-u)\right\}$$

$$t_{drg} - \bar{Y} = \bar{Y}\{e_y + (g_1 - g_2)\gamma e_{x_1} - (g_1 - g_2)\gamma e_{x_2} + \left[-\gamma^2 g_1 g_2 + g_2{}^2\left(\frac{\gamma^2+\gamma}{2}\right) + g_1{}^2\left(\frac{\gamma^2-\gamma}{2}\right)\right]e_{x_1}{}^2 + \left[-\gamma^2 g_1 g_2 + g_1{}^2\left(\frac{\gamma^2-\gamma}{2}\right) + g_2{}^2\left(\frac{\gamma^2+\gamma}{2}\right)\right]e_{x_2}{}^2 + [2\gamma^2 g_1 g_2 - \gamma^2(g_1^2 + g_2^2)]e_{x_1}e_{x_2} + (g_1 - g_2)\gamma e_y e_{x_1} - (g_1 - g_2)\gamma e_y e_{x_2}\}$$

$$\mathrm{B}(t_{drg}) = E(t_{drg} - \bar{Y}) = \bar{Y}\left\{\left[g_2{}^2\left(\frac{\gamma^2+\gamma}{2}\right) + g_1{}^2\left(\frac{\gamma^2-\gamma}{2}\right) - \gamma^2 g_1 g_2\right]\lambda' C_x^2 + \left[g_1{}^2\left(\frac{\gamma^2-\gamma}{2}\right) + g_2{}^2\left(\frac{\gamma^2+\gamma}{2}\right) - \gamma^2 g_1 g_2\right]\lambda C_x^2 + (2\gamma^2 g_1 g_2 - \gamma^2(g_1^2 + g_2^2))\lambda' C_x^2 + (g_1 - g_2)\gamma\lambda'\rho C_y C_x - (g_1 - g_2)\gamma\lambda\rho C_y C_x\right\} \quad \ldots (26)$$

The Mean Square error is given as:

$$MSE(t_{drg}) = \bar{Y}^2\left\{\lambda C_y^2 + 2\left(\frac{a^*-1}{1+a^*}\right)(\lambda-\lambda')\gamma\rho C_y C_x + \left(\frac{a^*-1}{1+a^*}\right)^2\gamma^2(\lambda-\lambda')C_x^2\right\} \quad \ldots (27)$$

For $a^* = 0$ and $\gamma = 1$ the estimator t_{drg} in (24) gives the classical ratio estimator in double sampling.

To obtain the optimal value of γ, differentiate (27) with respect to γ and set the resulting expression to zero. Thus,

$$\gamma = \frac{k}{g_1 - g_2} = k\frac{(1+a^*)}{(1-a^*)} \quad \ldots (28)$$

Putting (28) into (27) gives the optimum MSE as:

$$MSE_{opt}(t_{drg}) = \bar{Y}^2 C_y^2\{\lambda - (\lambda - \lambda')\rho^2\} \quad \ldots (29)$$

Remark 3: (29), is similar to the variance of the regression estimator of population mean in double sampling. Some members of the generalized estimator with their MSE for case I are shown in Table 3

Remark 4:
It should be noted that the optimal value of $'a'$ can also be obtained from (28) by making $'a'$ the subject of the formula. Thus;

$$a^* = \frac{\gamma-k}{\gamma+k} \quad \ldots (30)$$

3.3.1 Efficiency Comparison for the proposed generalized estimator

It should be recalled that the AOE of the generalized class of the suggested estimator has the same efficiency as the classical regression estimator in double sampling under case I is the same. Therefore the AOE is uniformly better than any other ratio estimator in double sampling, whose efficiency is not equal to or greater than the classical regression estimator. For other members of the proposed generalized family, a member say t_{drgi} is better than t_{drgj}

$$t_{drgj} \text{ iff } MSE(t_{drgi}) < MSE(t_{drgj}) \quad \ldots (31)$$

TABLE 3
Some members of the generalized estimators with MSE (Case I)

S/N	Estimators	MSEs
1.	$t_{drg1} = \bar{y}$, Sample Mean	$\bar{Y}^2\lambda C_y^2$
2.	$t_{drg2} = \bar{y}(\frac{\bar{x}_1}{\bar{x}_2})$, Sukhatme (1962)	$\bar{Y}^2[\lambda C_y^2 + (\lambda-\lambda')(C_x^2 - 2\rho C_y C_x)]$
3.	$t_{drg3} = \bar{y}(\frac{\bar{x}_1}{\bar{x}_2})^2$	$\bar{Y}^2[\lambda C_y^2 + 4(\lambda-\lambda')(C_x^2 - \rho C_y C_x)]$
4	$t_{drg4} = \bar{y}\sqrt{(\frac{\bar{x}_1}{\bar{x}_2})}$	$\bar{Y}^2[\lambda C_y^2 + (\lambda-\lambda')(\frac{C_x^2}{4} - \rho C_y C_x)]$
5	$t_{drg5} = \bar{y}(\frac{\bar{x}_1}{\bar{x}_2})^k$	$\bar{Y}^2[\lambda C_y^2 + k(\lambda-\lambda')(kC_x^2 - 2\rho C_y C_x)]$
6	$t_{drg6} = \bar{y}\left(\frac{\bar{x}_1 + \left(\frac{1-2k}{1+2k}\right)\bar{x}_2}{\left(\frac{1-2k}{1+2k}\right)\bar{x}_1 + \bar{x}_2}\right)^{\frac{1}{2}}$	$\bar{Y}^2[\lambda C_y^2 + k(\lambda-\lambda')(kC_x^2 - 2\rho C_y C_x)]$
7	$t_{drg7} = \bar{y}\left(\frac{\bar{x}_1 + \frac{1}{2}\bar{x}_2}{\frac{1}{2}\bar{x}_1 + \bar{x}_2}\right)^{3k}$	$\bar{Y}^2[\lambda C_y^2 + k(\lambda-\lambda')(kC_x^2 - 2\rho C_y C_x)]$

Case II

For case II of the proposed generalized estimator, the bias and mean square error can be obtained as:

$$B_2(t_{drg}) = \bar{Y}\left\{\left[\frac{a^2}{(1+a)^2}\left(\frac{\gamma^2+\gamma}{2}\right) + \frac{1}{(1+a)^2}\left(\frac{\gamma^2-\gamma}{2}\right) - \frac{a}{(1+a)^2}\gamma^2\right]\lambda' C_x^2 + \left[\frac{1}{(1+a)^2}\left(\frac{\gamma^2-\gamma}{2}\right) + \frac{a^2}{(1+a)^2}\left(\frac{\gamma^2+\gamma}{2}\right) - \frac{a}{(1+a)^2}\gamma^2\right]\lambda C_x^2 - \left(\frac{1-a}{1+a}\right)\gamma\lambda\rho C_y C_x\right\} \quad \ldots (32)$$

And the mean square error as:

$$MSE_2(t_{drg}) = \bar{Y}^2\left\{\lambda C_y^2 - 2\left(\frac{1-a}{1+a}\right)\lambda\gamma\rho C_y C_x + \left(\frac{1-a}{1+a}\right)^2\gamma^2(\lambda+\lambda')C_x^2\right\} \quad \ldots (33)$$

To obtain the optimum MSE, (33) is differentiated partially and the resulting expression is equated to zero, we get the optimum value of γ as :

$$\gamma = \frac{\lambda k}{(g_1-g_2)(\lambda+\lambda')} = \frac{\lambda k}{\left(\frac{1-a^*}{1+a^*}\right)(\lambda+\lambda')} = \frac{\lambda k(1+a^*)}{(1-a^*)(\lambda+\lambda')} \quad \ldots (34)$$

Putting (34) into equation (33) gives

$$MSE_{2_{opt}}(t_{drg}) = \bar{Y}^2 C_y^2\{\lambda - w\rho^2\} \quad \ldots (35)$$

where $w = \frac{\lambda^2}{(\lambda+\lambda')}$

Some members of this estimator in case II are similar to those of case I, except that for the AOE where the values of $'a^*'$ and γ differs significantly. Table 4 shows some members of the estimator in this case with their MSE.

TABLE 4
Some members of the generalized estimators with MSE (Case II)

S/N	Estimators	MSEs
1.	$t_{drg1} = \bar{y}$, Sample Mean	$\bar{Y}^2\lambda C_y^2$
2.	$t_{drg2} = \bar{y}(\frac{\bar{x}_1}{\bar{x}_2})$,Sukhatme (1962)	$\bar{Y}^2[\lambda C_y^2 + (\lambda+\lambda')C_x^2 - 2\lambda\rho C_y C_x]$
3	$t_{drg3} = \bar{y}\sqrt{(\frac{\bar{x}_1}{\bar{x}_2})}$	$\bar{Y}^2[\lambda C_y^2 + \frac{(\lambda+\lambda')}{4}C_x^2 - \lambda\rho C_y C_x]$
4	$t_{drg4} = \bar{y}(\frac{\bar{x}_1}{\bar{x}_2})^{\frac{\lambda k}{\lambda+\lambda'}}$,	$\bar{Y}^2[\lambda C_y^2 + \frac{\lambda^2k^2(\lambda+\lambda')}{(\lambda+\lambda')^2}C_x^2 - 2\frac{\lambda^2 k}{(\lambda+\lambda')}\rho C_y C_x]$
5	$t_{drg5} = \bar{y}\left(\frac{\bar{x}_1 + \frac{1}{2}\bar{x}_2}{\frac{1}{2}\bar{x}_1 + \bar{x}_2}\right)^{3k}$	$\bar{Y}^2[\lambda C_y^2 + (\lambda+\lambda')k^2C_x^2 - 2\lambda k\rho C_y C_x]$
6	$t_{drg6} = \bar{y}(\frac{\bar{x}_1}{\bar{x}_2})^k$	$\bar{Y}^2[\lambda C_y^2 + (\lambda+\lambda')C_x^2 - 2\lambda k\rho C_y C_x]$
7	$t_{drg7} = \bar{y}\left(\frac{\bar{x}_1 + \frac{1}{2}\bar{x}_2}{\frac{1}{2}\bar{x}_1 + \bar{x}_2}\right)^{\frac{3\lambda k}{(\lambda+\lambda')}}$	$\bar{Y}^2[\lambda C_y^2 + \frac{\lambda^2k^2(\lambda+\lambda')}{(\lambda+\lambda')^2}C_x^2 - 2\frac{\lambda^2 k}{(\lambda+\lambda')}\rho C_y C_x]$

3.3 Numerical Application

In order to validate the theoretical claims of this research work empirical results are employed using the data obtained from some existing work as shown in Table 5. MSE of existing ratio estimators and the suggested generalized ratio estimator for case I and II are presented in Table 6, Table 7, and Table 8 respectively.

Sources of Populations:
Population I: Cingi [2007]
Population II : Murthy [1967]

Population III: Kardilar & Cingi [2006]
Population IV: Handique [2012]

TABLE 5
Populations and Their Parameters

populations	N	n_1	n_2	$\bar{Y}$	$\bar{X}$	C_y	C_x	ρ
I	923	400	200	436.3	11440.5	1.72	1.86	0.955
II	80	30	10	5182.64	1126.46	0.35426	0.75067	0.9413
III	104	40	20	625.37	13.93	1.866	1.653	0.865
IV	2500	200	25	4.63	21.09	0.95	0.98	0.79

TABLE 6
Mean square errors of some existing ratio estimators in two phase sampling

Estimators	Populations			
	I	II	III	IV
$\bar{y}$, Sample mean	2,196.30	294,954.05	55,014.70	0.766
Ratio	934.83	407,572. 37	29,557.12	0.383
Singh & Vishwakarma(2007)	1153.96	98,876.96	35,607.08	0.395
Regression	912.27	95,736.20	29,542.31	0.344

TABLE 7
MSE of some members of the generalized class of estimators (Case I)

Estimators	Populations			
	I	II	III	IV
t_{drg1}	2,196.30	294,954.1	55,014.70	0.766
t_{drg2}	934.83	407,572.37	29,557.12	0.383
t_{drg3}	2,966.04	2,539,283.15	57,530.10	1.441
t_{drg4}	1,153.93	98,876.96	35,607.08	0.395
t_{drg5}	912.27	95,736.20	29,542.31	0.344
t_{drg6}	912.27	95,736.20	29,542.31	0.344
t_{drg7}	912.27	95,736.20	29,542.31	0.344

TABLE 8
MSE of some members of the generalized estimators (Case II)

Estimators	Populations			
	I	II	III	IV
$t_{drg1}^{\square}$	2,196.30	294,954,05	55,014.70	0.766
$t_{drg2}^{\square}$	1,150.29	757,513.14	30,331.86	0.427
$t_{drg3}^{\square}$	800.703	116,436.69	27,766.15	0.369
$t_{drg4}^{\square}$	722.33	83,804.54	25,211.82	0.337
$t_{drg5}^{\square}$	912.27	95.729.74	29.543.56	0.343
$t_{drg6}^{\square}$	912.27	95,729.74	29,543.56	0.343
$t_{drg7}^{\square}$	722.33	83,804.54	25,211.82	0.337

4. Discussion of Results

In this work, a class of ratio estimators in two phase sampling scheme is suggested as shown in [1]. Some members of the proposed estimator are shown in Table 2. Suitable values of the scalar $'a'$ in the suggested estimator gave rise to different members of the suggested estimators. More so, its bias and mean square error in both cases were obtained as shown in [6], [8] and [12], [13] respectively. The optimality condition for both cases was obtained as expressed in [9] and [14] respectively giving rise to the optimal mean square error for both cases as shown in [10] and [15] respectively. Just as in Singh and Choudhury [2012] estimator, it was discovered that the AOE in the advocated estimator in case I has the same efficiency as the conventional regression estimator in two phase sampling. The conditions for which the AOE of this class of estimators would be uniformly better than the estimators of Sukhatme [1962], Singh and Vishwarkama [2007] and the simple random sample mean was obtained. It was observed that the suggested estimator at optimal condition was uniformly better, than some of the existing estimators shown in Table 1; Sukhatme [1962], Singh and Vishwarkama [2007] and the simple random sample mean. Also a condition for a member of this class to be more efficient than another member was also established.

Furthermore, a generalized form of the suggested estimator was obtained as shown in [24]. In a similar manner, the bias and MSE of the generalized estimator was obtained for cases I and II as expressed in [26], [27] and [32], [33] respectively. The optimality condition for both cases is as shown in [28] and [34] respectively. Also the optimum mean square error for both cases is as expressed in [29] and [35] respectively. It was observed that the efficiency of the AOE and the conventional regression estimator in double sampling was the same in case I. However the AOE performed better that the classical regression estimator in double sampling in case II.

Most importantly, it was observed from Tables 2 and 3 that the advocated generalized estimator had more than one AOE as the values of the scalars $'a'$ and γ was varied in [24]. This means that some members of this generalized estimator have the same minimum mean square error.

Four populations and parameters as shown in Table 4 were used for empirical validation of the theoretical results of this work. From Tables 7 and 8, it was discovered that t_{drg5}^{I}, t_{drg6}^{I} and t_{drg7}^{I} had the least MSEs for case I, while t_{drg4}^{II} and t_{drg7}^{II} had the least MSEs for case II. Therefore, in terms of efficiency, these estimators were adjudged to be uniformly better than other members of the family as well as the existing estimators shown in Table 1 and 6.

Further observation revealed that the AOEs t_{drg5}^{I}, t_{drg6}^{I} and t_{drg7}^{I} in case I have the smallest mean squares error in the four populations as shown in Table 7, while the AOEs t_{drg4}^{II} and t_{drg7}^{II} of the suggested class of estimators in case II have the smallest mean square error in the four populations as shown in Table 8. However, the AOEs in case II where sub-sampling was done independent of the first phase sample performed better than in case I.

5. CONCLUSION

In summary the Asymptotic Optimum Estimators in both cases of double sampling shows greater gain in efficiency than some existing estimators mentioned in this work.

This research work suggested a family of ratio estimator with its generalized form in two phase sampling techniques with significant gain in efficiency at optimal condition. This gain in efficiency is more significant when sampling in the second phase does not depend on the first phase. More so, the suggested estimator performs better in case II than the regression estimator under two phase sampling.

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